

2111000303023001
EXAMINATION OCTOBER 2024
BACHELOR OF SCIENCE (COMPUTER SCIENCE)
(NON-NEP) (THIRD SEMESTER)
MATHEMATICS PAPER-V (MTH-301) (OLD)

[Time: As Per Schedule]

[Max. Marks : 50]

Instructions:

1. Fill up strictly the following details on your answer book

- a. Name of the Examination: **BACHELOR OF SCIENCE (COMPUTER SCIENCE) (NON-NEP) (THIRD SEMESTER)**
 - b. Name of the Subject: **MATHEMATICS PAPER - V (MTH-301) (OLD)**
 - c. Subject Code No: **2111000303023001**
2. Sketch neat and labelled diagram wherever necessary.
3. Figures to the right indicate full marks of the question.
4. All questions are compulsory.

Seat No:

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Student's Signature

Q.1 Answer the following (any five)

10

1. Define Jacobian of a bivariate function.
2. If $f_{xx}(a, b) = r, f_{xy}(a, b) = s, f_{yy}(a, b) = t$ then write down your conclusions regarding the different values of $rt - s^2$
3. Obtain $\lim_{x \rightarrow 0} \left\{ \lim_{y \rightarrow 0} \frac{x^2 - y^2}{x^2 + y^2} \right\}$ and $\lim_{y \rightarrow 0} \left\{ \lim_{x \rightarrow 0} \frac{x^2 - y^2}{x^2 + y^2} \right\}$.
4. Find degree of a homogeneous function $f(x, y) = \frac{x^3 + y^3}{x + y}$
5. If $f = x^2 + y^2 - 3xyz$, then find the value of $grad f$.
6. Obtain U_y and U_z for $U(x, y, z) = \sqrt{x^2 + y^2 + z^2}$.
7. If $\vec{r} = (1 - sint)\hat{i} + (t + cost)\hat{j} + (t^3 + t^2 + t + 1)\hat{k}$ then find $\frac{d\vec{r}}{dt}$.
8. If $x = u(1 + v), y = v(1 + u)$ then find $J(x, y)$.

Q.2 Answer the following (Any two)**10**

1. If f is a differentiable homogeneous function in x and y of degree m then prove that $xf_x + yf_y = mf$.
2. If $u = \log(x^2 + y^2 + z^2)$, then prove that $x \frac{\partial^2 u}{\partial y \partial z} = y \frac{\partial^2 u}{\partial z \partial x} = z \frac{\partial^2 u}{\partial x \partial y}$
3. Discuss the continuity of the function $f(x, y)$ at the point $(0, 0)$ where
$$f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}, (x, y) \neq (0, 0)$$
$$= 2, (x, y) \neq (0, 0)$$

Q.3 Answer the following (any two):**10**

1. Obtain expression of $e^{ax} \cos by$ in the form of powers of x and y .
2. If $x + y + z = u$, $y + z = uv$, $z = uvw$ then obtain $J(x, y, z)$ with respect to u, v, w .
3. Expand $f(x, y) = \frac{y^2}{x^3}$ in the powers of $(x - 1)$ and $(y + 1)$

Q.4 Answer the following (Any two)**10**

1. Find extreme values of $f(x, y) = x^3 + y^3 - 3x - 12y + 5$.
2. Show that $f(x, y) = 2(x - y)^2 - x^4 - y^4$ is maximum at $(\sqrt{2}, -\sqrt{2})$. Also find the maximum value.
3. Find three positive real numbers whose sum is 24 and product is maximum.

Q.5 Answer the following (Any two)**10**

1. Define divergence of a vector function. If \vec{U} and \vec{V} are differentiable vector functions, then prove that $div(\vec{U} + \vec{V}) = div \vec{U} + div \vec{V}$
2. If c is a triangle formed by vertices $(1, 0)$, $(0, 1)$ and $(-1, 0)$ then find $\int_c (y^2 dx - x^2 dy)$.

3. If $\vec{f} = (x^2yz)\hat{i} + (xy^2z)\hat{j} + (xyz^2)\hat{k}$, then find $\text{div}(\text{Curl } \vec{f})$.
